Indian Statistical Institute, Bangalore

B. Math (Hons.) Second Year

First Semester - Group Theory

Back Paper Exam — Date: 24th December 2024

Total marks: 100 — Duration: 3 hours

Each question carries 20 marks and Answer any 5.

1. (a) For any homomorphism T between groups, show that $T(e) = e$ *(Marks: 6)*.

   (b) Let H and K be subgroups of G. Prove that HK is a subgroup of G if and only if $HK = KH$

2. (a) Prove that a finite group G contains an element of order p if p is a prime number dividing $o(G)$ *(Marks: 8)*.

   (b) Prove that there are exactly two groups of order 6 up to isomorphism.

3. (a) Find the smallest subgroup of S_n containing $(1, 2)$ and $(1, 2, \cdots, n)$.

   (b) Prove that permutations acting on disjoint sets commute *(Marks: 10)*.

4. (a) Prove $\{(g, g) \mid g \in G\}$ is normal in $G \times G$ if and only if G is abelian.

   (b) Find a necessary and sufficient condition on groups G and H so that $G \times H$ is cyclic *(Marks: 12)*.

5. (a) Describe all finite abelian groups of order 2^4.

   (b) Determine conjugacy classes in SU_2 *(Marks: 12)*.

6. (a) State and prove third sylow theorem *(Marks: 14)*.

   (b) Find the conjugacy classes and centralizers of each permutation in S_3.